Tunneling through a fluctuating barrier: Two-level model

Jan Iwaniszewski*
School of Physics and Chemistry, Lancaster University, Lancaster, LA1 4YB, United Kingdom
and Institute of Physics, Nicholas Copernicus University, Grudziądzka 5, 87-100 Toruń, Poland†

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We investigate the problem of tunneling across a randomly fluctuating barrier in the presence of dissipation in the two-level approximation. The barrier fluctuations are induced by a random telegraph noise whose switching rate ν is taken as a control parameter. For infinitely fast fluctuations the dynamics of the system is similar to the static case, while, for very small ν, the barrier evolution is a superposition of static solutions for both configurations. This leads to a resonant beating or long-time periodic localization. For an intermediate value of ν we have found a resonancelike suppression of coherent tunneling. When the system levels are detuned, a resonant enhancement of decay in the incoherent regime also occurs.

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I. INTRODUCTION

The problem of relaxation from a metastable state in a double-well potential occurs in many problems in physics and chemistry as well as in other scientific areas [1]. In classical systems the process occurs due to the ubiquity of fluctuations, not necessarily of thermal origin, and one must consider the problem in the framework of dissipative dynamics. In quantum systems a metastable state may be also emptied due to the tunneling effect, so the problem appears as a competition between coherent quantum dynamics and incoherent dissipation. The simplest model of a quantum particle moving in a double-well potential is given by a two-level system (TLS) [2]. This represents a reasonable approximation when the lowest two states of the system are well localized in the two potential wells, and their energies are much less than the energy of higher states as well as than the barrier height. Within this approximation an isolated system is completely described by two parameters only: the tunneling matrix element Δ which couples the levels and contains information about the height of the barrier; and the detuning parameter ϵ, i.e., the difference between the energies of the ground states of the wells, which accounts for the asymmetry in the system. Although this formulation seems very simple of first sight, the coupling to the thermal bath substantially complicates the problem [2,3].

An important generalization appears when the tunneling system is exposed to time-dependent external fields [4,5]. This is the case when one drives the quantum system with a strong laser field which results in modulation of the bias parameter ϵ. Until now the latter topic has attracted the majority of the attention: only a few papers have considered the problem of a time-dependent Δ, i.e., the effect of barrier modulation on the tunneling dynamics. Grifoni and co-workers [6,7,5] drive the barrier with a periodic signal, while Goychuk and co-workers [8–11] used a stochastic perturbation of Δ.

In this article we also address the problem of randomly driven barrier height in the TLS approximation. Our main interest is the dependence of the population dynamics on the correlation time τ of the barrier fluctuations. The corresponding problem has been considered in classical systems, leading to the discovery of resonant activation [12]—the appearance of a maximal value of the mean escape rate for some finite value of τ. Further, it has been proved that an opposite effect, called inhibition of activation, may also occur [13], i.e., a maximal slowing down of the activation process for some finite degree of correlation of the barrier perturbation. Our aim in what follows is to look for similar resonancelike effects in quantum systems. Goychuk and co-workers initialized studies of this problem. In Ref. [9] they found both numerically and within certain approximations that the transfer rate in dissipative tunneling approaches a maximum for τ^{-1} of the order of unperturbed Δ. They also considered [8] the influence of τ on the transition from coherent to incoherent evolution, although only in a degenerate case ($\epsilon=0$) and for some particular examples. Here we treat this problem in a more systematic way, looking for any resonancelike features in the time dependence of the system.

The physical situation which we intend to investigate is known to occur, e.g., in long-range electron transfer reactions [14,8,11]. In such reactions the tunneling distance extends up to $20-30$ Å, much more than the range of the overlap of atomic orbitals of the donor and the acceptor. The state of the medium between those centers (the bridge) plays an essential role in the transfer process, so the dynamics of conformational variations of the molecule cannot be ignored. Another area where the stochastic perturbation affects the tunneling barrier is semiconductor physics (e.g., Ref. [15]), where lattice excitations permanently modify the shape of the band structure. An external random perturbation of a barrier can also arise when some control parameters fluctuate, e.g., the electric field used to polarize a multilayer structure.

The problem of tunneling through a time-modulated barrier is known also in the scattering area. Some authors [16,17] analyzed this phenomenon in the presence of periodically oscillating height of the barrier. The main difference between that case and the present one stems from the form of the potential. In the scattering problems the potential is of a finite range, outside which the particle moves freely. Hence

*Electronic address: jan.iwaniszewski@phys.uni.torun.pl
†Permanent address.

the system is characterized by continuum of possible states with some resonant structure induced by the barrier modulation [16]. The particle interacts with the barrier during a finite time interval, being reflected or transmitted with a simultaneous loss or gain of quantum of energy of the oscillations. Our model concerns a confined system with a well-defined number of discrete states; consequently one can speak about a localization of the tunneling particle or about a quantum coherence and its suppression.

As already mentioned, the TLS approximation of the tunneling problem can legitimately be applied only when certain relations between the characteristic energies of the system are fulfilled. This in turn imposes restrictions on the possible values of the parameters used. However, in what follows we will not comply with them because our two-level model may also describe other quantum systems where there are no such constraints on the parameters, e.g., a two-level atom interacting with an electromagnetic field.

The rest of the paper is organized as follows. In Sec. II we describe our system and obtain basic dynamical equations. In Sec. III we review the properties of a static TLS system, and consider the two limiting cases of infinitely fast and slow fluctuations. Next we present the exact results for the unbiased case (Sec. IV) and some approximations and numerical results for the biased case (Sec. V). The results obtained, and the conclusions drawn are presented and discussed in Sec. VI.

II. SYSTEM

The Hamiltonian of the system can be written as

$$H=-\frac{1}{2}\hbar\Delta(t)\sigma_x+\frac{1}{2}\hbar\epsilon\sigma_z+\frac{1}{2}\hbar\xi(t)\sigma_z. \tag{1}$$

The Pauli matrices σ_i $(i=x,y,z)$ are the basis operators in the localized representation in which the eigenstates of σ_z correspond to localization of the system in one of the potential wells. The first term in Eq. (1) describes tunneling between the wells. Due to the barrier fluctuations the tunneling matrix element $\Delta(t)$ is a random function of time. In the following discussion we assume that these fluctuations arise from a symmetric dichotomic noise (DN) $\eta(t)$. Exploiting the properties of DN independently of the way it acts on the barrier, and without any loss of generality, one may write a decomposition $\Delta(t)=\Delta_0+\Delta_1\eta(t)$. The noise $\eta(t)$ of zero mean is characterized by correlation function $\langle\eta(t)\eta(s)\rangle=\exp(-2\nu|t-s|)$, where the jump rate ν is half of the inverse of the correlation time τ of this noise.

The second term in H comes from the detuning energy $\hbar\epsilon$ between the two levels, while the last one gives the interaction with the environment. It is given by a simple bilinear coupling of the operator σ_z, which is the quantum counterpart of position for a tunneling particle, with a zero-mean Gaussian white noise $\xi(t)$, which represents the environment. This noise is parametrized by the intensity 2κ, while its correlation function reads $\langle\xi(t)\xi(s)\rangle=4\kappa\delta(t-s)$. Such a model of interaction with the surroundings may be considered as a high temperature approximation of the quantum oscillators model of a thermal bath [18,3]. Using this model we neglect many interesting features related to the quantum nature of the environment [2,3]. However, our aim here is to study the influence on the tunneling effect of a random disturbance of the potential barrier, whereas the existence of the environment just plays a secondary role. We need it only to introduce a mechanism of dissipation.

Suppose that at $t=0$ the particle is localized in one of the potential wells (say the right-hand one). We are interested in the probability $P(t)$ of finding the particle in this well at times $t\geqslant 0$. This is simply related to the mean value of σ_z:

$$P(t)=\frac{1}{2}(\langle\sigma_z(t)\rangle+1). \tag{2}$$

Here $\langle\cdots\rangle$ means averaging over both noises $\xi(t)$ and $\eta(t)$ as well as over quantum degrees of freedom. Following Refs. [19] or [20] to obtain the von Neuman–Liouville equation for the density operator of the system, and then exploiting the Shapiro-Loginov theorem [21], one comes to a set of six linear ordinary differential equations which completely describes the system,

$$\frac{d\vec{R}}{dt}=A\vec{R}, \tag{3a}$$

where

$$\vec{R}=\begin{pmatrix}\langle\sigma_x(t)\rangle\\ \langle\sigma_y(t)\rangle\\ \langle\sigma_z(t)\rangle\\ \langle\sigma_x(t)\eta(t)\rangle\\ \langle\sigma_y(t)\eta(t)\rangle\\ \langle\sigma_z(t)\eta(t)\rangle\end{pmatrix}, \tag{3b}$$

$$A=\begin{pmatrix}-2\kappa & -\epsilon & 0 & 0 & 0 & 0\\ \epsilon & -2\kappa & \Delta_0 & 0 & 0 & \Delta_1\\ 0 & -\Delta_0 & 0 & 0 & -\Delta_1 & 0\\ 0 & 0 & 0 & -2\kappa-2\nu & -\epsilon & 0\\ 0 & 0 & \Delta_1 & \epsilon & -2\kappa-2\nu & \Delta_0\\ 0 & -\Delta_1 & 0 & 0 & -\Delta_0 & -2\nu\end{pmatrix}. \tag{3c}$$

The problem has five parameters, four of which—ϵ, Δ_1, κ, and ν—may be equal to 0. Only $\Delta_0\neq 0$, since we do consider a tunneling problem. (Note that because of this Δ_1 cannot be greater than Δ_0.) From the form of the evolution matrix A [Eq. (3c)], it follows that only the relative values of the previous four parameters with respect to Δ_0 will be important for the evolution. For the convenience of further discussion we leave all the quantities in the formulas below, but in numerics we use such relative values of parameters (simply $\Delta_0=1$). Also time in the figures is given in dimensionless units $t\rightarrow t/\Delta_0$.

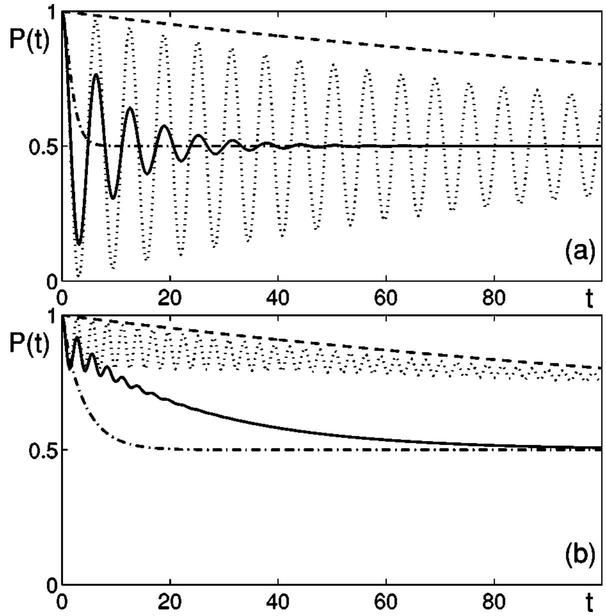

FIG. 1. Time evolution of $P(t)$ for a static barrier ($\Delta_1=0$), with $\Delta\equiv\Delta_0=1$ for the unbiased case with $\epsilon=0$ (a) and the biased one with $\epsilon=2$ (b). The damping rate κ induced by the environment reads 0.01 (dotted curve), 0.1 (continuous curve), 1.0 (dash-dotted curve), and 100 (dashed curve). In all figures the dimensionless time is given in units of Δ_0.

III. SOME SPECIAL CASES

A. Static barrier

Before we will investigate the influence of barrier fluctuations on the tunneling process, let us first briefly recall the main features of tunneling through a static barrier [2,22,3,7]. In a symmetric case ($\epsilon=0$), and without any interaction with the surroundings, $P(t)$ oscillates continuously with frequency Δ and amplitude 1/2. Coupling to the thermal bath impedes tunneling, which results in a damping of the coherent evolution,

$$P(t)=\frac{1}{2}\left[1+\frac{\Delta}{\Omega}\exp(-\kappa t)\cos(\Omega t-\phi)\right], \tag{4}$$

where $\sin(\phi)=\kappa/\Delta$, as well as in a decrease of the frequency of the oscillations

$$\Omega=\sqrt{\Delta^2-\kappa^2}. \tag{5}$$

When κ becomes greater than Δ the evolution becomes completely incoherent,

$$P(t)=\frac{1}{2}+\frac{1}{4\Gamma}[(\Gamma+\kappa)\exp[-(\kappa-\Gamma)t] + (\Gamma-\kappa)\exp[-(\kappa+\Gamma)t)], \tag{6}$$

where $\Gamma=\sqrt{\kappa^2-\Delta^2}$. A further increase of κ slows down the tunneling so much that the system stays in the initial well for a very long time, which is known as localization induced by damping [2]. Some examples of the evolution of $P(t)$ are displayed in Fig. 1(a).

A nonzero bias ($\epsilon\neq 0$) also makes it more difficult to tunnel. For $\kappa=0$, we have

$$P(t)=\frac{1}{2}\left[1+\frac{\epsilon^2}{\Omega^2}+\frac{\Delta^2}{\Omega^2}\cos(\Omega t)\right], \tag{7}$$

where

$$\Omega=\sqrt{\Delta^2+\epsilon^2}. \tag{8}$$

Although ϵ speeds up the oscillations, it also decreases their amplitude, so that less than the entire amount of probability is sent between the levels. If $\kappa\neq 0$ one has to solve a three-dimensional problem. Although this may be done analytically, we do not present the result because it is terribly complicated. We mention here only that if $\epsilon^2<\Delta^2/8$ and $\kappa_2<\kappa<\kappa_2$, where

$$\kappa_{1,2}^2=\frac{\Delta^2}{8\varepsilon^2}[1+20\varepsilon^2-8\varepsilon^4\pm(1-8\varepsilon^2)^{3/2}], \quad \varepsilon=\epsilon/\Delta, \tag{9}$$

all three eigenvalues of the problem are real (negative). In the other cases there is one real eigenvalue and a pair of complex-conjugated eigenvalues, so the system exhibits damped oscillations. However, for large κ or ϵ the amplitude of these oscillations is negligible and the system relaxes incoherently to 1/2. Some examples of the evolution of $P(t)$ are presented in Fig. 1(b). As for the unbiased case, the coupling to the bath suppresses the coherence but, because of the asymmetry, the center of oscillations deviates toward $P>1/2$. On the other hand, an interaction with the surroundings also unloads the surplus of probability of the initial state, so eventually P is distributed equally between both levels.

B. Infinitely fast fluctuations

In the limit $\nu\to\infty$ the auxiliary quantities $\langle\sigma_i(t)\eta(t)\rangle$ relax to zero within the infinitely fast scale of time, so the system evolves as in the static case with $\Delta=\Delta_0$. In other words, for very fast fluctuations of $\Delta(t)$ the dynamics is governed by its mean value Δ_0.

C. Infinitely slow fluctuations

It follows from a simple analysis of Eqs. (3a)–(3c) that, in the limit of very slow fluctuations ($\nu\to 0$) the evolution of the system is a superposition of the evolutions in two static configurations with tunneling matrix elements $\Delta_\pm=\Delta_0\pm\Delta_1$, respectively. Consequently, the evolution of $P(t)$ is richer than previously, with different combinations of types of solutions for static cases with Δ_+ and Δ_-. Some examples are displayed in Figs. 2 and 3. For the unbiased case ($\epsilon=0$) when κ is small $P(t)$ relaxes with two-frequency oscillations. For small Δ_1 these frequencies are similar (see Ref. [5]), and we observe a beating phenomenon [Fig. 2(a)]. On the other hand, when Δ_1 is large, so that Δ_- is much smaller than Δ_+, these frequencies are correspondingly very different, and one can notice a phenomenon similar to localization in a potential well—for half of the period of slower oscillations the system ''prefers'' to stay in one of the potential wells exhibiting only fast oscillations (the larger frequency) inside it [Fig. 3(a)].

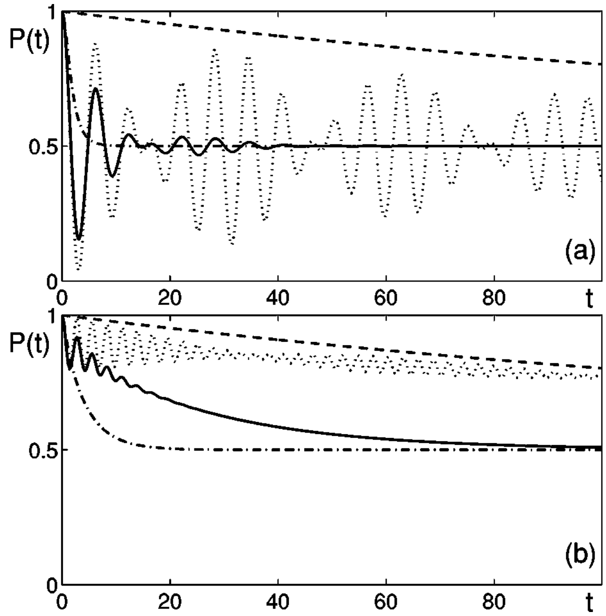

FIG. 2. Time evolution of $P(t)$ for an infinitely slow ($\nu=0$) random perturbation of the barrier with a small amplitude ($\Delta_1=0.1$). The other parameters are the same as in Fig. 1.

An increase of κ damps the oscillations and leads to incoherent tunneling. However, if Δ_1 is comparable with Δ_0 the difference between Δ_+ and Δ_- is large, so that the critical values of κ for a transition from coherent to incoherent dynamics in the two configurations are very different. Consequently one can observe single-frequency damped oscillations moved toward $P>1/2$, an effect very similar to that of the biased static barrier in Fig. 1(b). However, the reason for the deviation is now different. It occurs due to the slowing down of relaxation in the Δ_- configuration.

As in Sec. III A, when $\epsilon\neq 0$, curves $P(t)$ are moved upwards [Figs. 2(b) and 3(b)]. We may note also that, since ϵ increases the frequency of oscillation in the coherent regime [see Eq. (8)], the beat frequency decreases, as can be seen in Fig. 2(b) where $\Delta_1=0.1$. In contrast, this effect is invisible when Δ_1 is large [Fig. 3(b)] because, as follows from Eq. (7), the amplitude of slow oscillations is very small. Coherent tunneling means single-frequency damped oscillations.

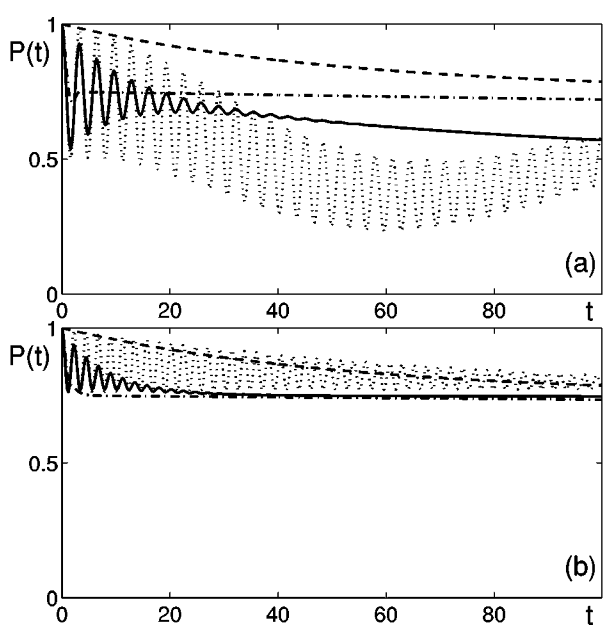

FIG. 3. The same as in Fig. 2, but for large amplitude ($\Delta_1=0.95$) of barrier fluctuations.

IV. UNBIASED CASE

For a system with degenerate energy levels ($\epsilon=0$) the quantities $\langle\sigma_x(t)\rangle$ and $\langle\sigma_x(t)\,\eta(t)\rangle$ are decoupled from other dynamical variables, and problem (3a) reduces to a fourth-dimensional one which, fortunately, is solvable analytically.

A. $\kappa=0$

Let us begin the analysis from the case when the system does not interact with the environment, i.e., when $\kappa=0$. If $\nu<\Delta_1$ the relevant eigenvalues λ of matrix A read

$$\lambda=-\nu\pm i\Omega_{1,2}, \tag{10}$$

where $\Omega_{1,2}=\Delta_0\pm\sqrt{\Delta_1^2-\nu^2}$, and

$$P(t)=\frac{1}{2}+\frac{\Delta_1}{4\sqrt{\Delta_1^2-\nu^2}}\exp(-\nu t)\times[\cos(\Omega_1 t+\phi)+\cos(\Omega_2 t-\phi)], \tag{11}$$

where $\sin(\phi)=\nu/\Delta_1$. As discussed in Sec. III C, if $\nu=0$ then $P(t)$ is a superposition of two coherent oscillations of frequencies $\Omega_{1,2}=\Delta_0\pm\Delta_1$, respectively. A nonzero value of ν brings these frequencies closer and causes damping. It also introduces a phase difference between the two modes and modifies their amplitudes. When ν approaches Δ_1 both frequencies approach Δ_0 and the phase difference equals $\pi/2$.

If $\nu>\Delta_1$ we have

$$\lambda=-\Gamma_{1,2}\pm i\Delta_0, \tag{12}$$

where $\Gamma_{1,2}=\nu\pm\sqrt{\nu^2-\Delta_1^2}$, and

$$P(t)=\frac{1}{2}+\frac{1}{2}\cos(\Delta_0 t)\left[\frac{1}{2}\left(1-\frac{\nu}{\sqrt{\nu^2-\Delta_1^2}}\right)\exp(-\Gamma_1 t)+\frac{1}{2}\left(1+\frac{\nu}{\sqrt{\nu^2-\Delta_1^2}}\right)\exp(-\Gamma_2 t)\right], \tag{13}$$

i.e., $P(t)$ is a superposition of two damped oscillating modes with the same, independent from ν, frequency. An increase of ν yields that the first mode (with Γ_1) is damped more and more strongly, and its amplitude decreases faster and faster toward 0 as $\nu\to\infty$. It is this mode which is associated with the fast time scale mentioned in Sec. III B. On the other hand, when ν increases, the second mode is damped less and less and its amplitude tends to unity. In the limit $\nu\to\infty$ this mode reaches an undamped oscillating state in accordance with Sec. III B.

It follows from the above discussion that although no damping originates from the interaction with the thermal bath ($\kappa=0$), nevertheless the system is damped for a finite value of fluctuating rate. This is because of the stochastic

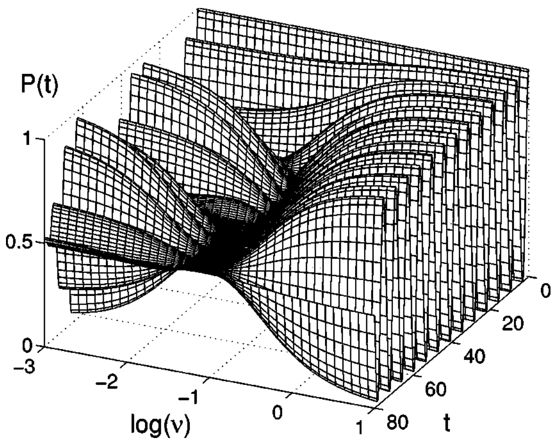

FIG. 4. Time evolution of $P(t)$ in dependence on the decimal logarithm of ν for the unbiased case ($\epsilon=0$) without interaction with the environment ($\kappa=0$), for $\Delta_1=0.1$.

time dependence of the tunneling matrix element, which dephases itself at random for any realization of the tunneling process, causing decoherence. An average over the whole ensemble of realizations of $\eta(t)$ results in an exponential damping of coherent oscillations as well as in alteration of amplitudes and relative phases of possible modes of $P(t)$. If $\nu<\Delta_1$, both modes are of equal importance, whereas for ν greater than Δ_1 one of them dominates the other. The strongest damping of the dominant oscillations appears for $\nu=\Delta_1$, which constitutes a resonancelike condition for the suppression of coherence in the system. Since neither quantities ν and Δ_1 concern the system itself but relate, rather, to the barrier disturbance, one may say that the perturbation is in resonance with itself. As a result of this resonance, one observes a maximal suppression of coherence in a tunneling process. An illustration of this effect is shown in Fig. 4 on a three-dimensional plot of the time evolution of $P(t)$ in dependence on ν.

B. $\kappa\neq 0$

If the system interacts with its surroundings the eigenvalues of A reads as [23]

$$\lambda=-\nu-\kappa\pm\sqrt{a\pm 2\sqrt{b}}, \tag{14}$$

where

$$a=\kappa^2+\nu^2-\Delta_0^2-\Delta_1^2, \tag{15a}$$

$$b=\kappa^2\nu^2+\Delta_0^2\Delta_1^2-\nu^2\Delta_0^2. \tag{15b}$$

[The two remaining irrelevant eigenvalues, which are associated with $\langle\sigma_x(t)\rangle$ and $\langle\sigma_x(t)\,\eta(t)\rangle$, are: $-\kappa$ and $-\kappa-\nu$.] Analyzing the signs and the relationship between a and b, one can distinguish four possibilities: (I) two pairs of complex conjugated eigenvalues with different real parts and the same imaginary ones; (II) two pairs of complex conjugated eigenvalues with the same real parts and different imaginary ones; (III) two different negative real eigenvalues and one pair of complex conjugated eigenvalues; and (IV) four different negative real eigenvalues. In Fig. 5 we illustrate the location of these types of eigenvalues in the parameter space

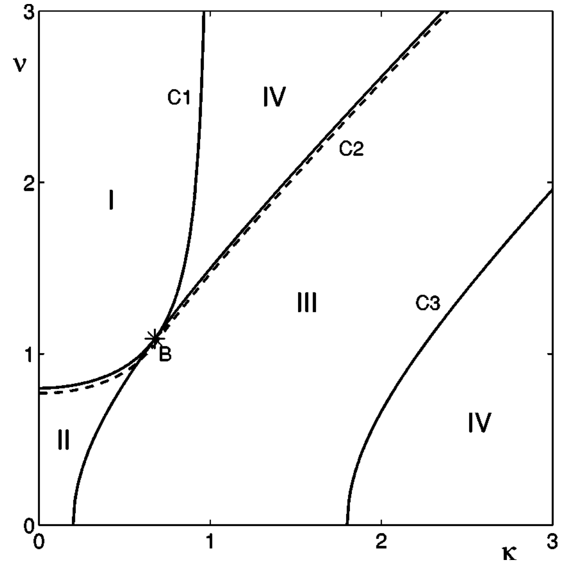

FIG. 5. Regions of different types of eigenvalues of matrix A for the biased system ($\epsilon=0$) with $\Delta_0=1.0$ and $\Delta_1=0.8$. Only the four eigenvalues relevant to this case are considered. The regions and curves are numbered in accordance with the text. The broken line mentioned in the text should follow C_1 until the point B, and then follow C_2.

(κ,ν). The curves C_i $(i=1,2,3)$ which separate the different regions are given by the expressions

$$C_1:\quad \nu=\Delta_0\Delta_1/\sqrt{\Delta_0^2-\kappa^2},$$

$$C_2:\quad \nu=\sqrt{(\kappa-\Delta_1)^2-\Delta_0^2},$$

$$C_3:\quad \nu=\sqrt{(\kappa+\Delta_1)^2-\Delta_0^2}.$$

One may note that any variation of Δ_1 only moves the boundaries between these regions. It has no effect on the topology of this figure.

Figure 5 confirms an obvious expectation that, for small κ, the evolution of $P(t)$ should be very similar to that which we discussed in Sec. IV A. Of course a nonzero value of κ modifies the frequency, but more important is that it increases the damping parameters, so the coherence of the tunneling process is being suppressed faster.

Relating our discussion to Sec. III C, it is also obvious that for small nonzero values of ν the system behaves very similarly to the case of infinitely slow fluctuations. As previously, a finite value of ν modifies the frequencies and increases damping rates. An increase of ν also modifies the boundaries between the three different types of evolution in such a way that they are moved toward greater values of κ. For larger ν the two-oscillatory region disappears, but the remaining two exist always.

The situation is a little bit different for very large ν. The expectation about the similarity to the dynamics of the case with infinitely fast fluctuations (Sec. III B) proves correct for finite κ only. For large κ the two regions mentioned above appear. We can also note that in comparison with the properties of the system in the infinitely slow limit, the main result of a finite value of ν is an increase of damping rates of the relevant modes.

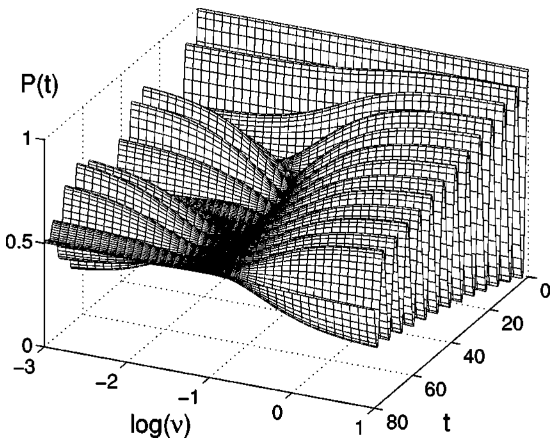

FIG. 6. Time evolution of $P(t)$ in dependence on the decimal logarithm of ν for the unbiased case ($\epsilon=0$) with weak interaction with the environment ($\kappa=0.01$), for $\Delta_1=0.1$.

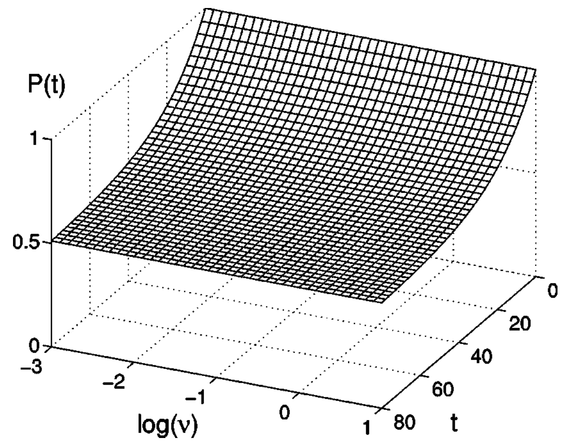

FIG. 7. The same as in Fig. 6, but for strong interaction with the environment ($\kappa=10$).

In light of this discussion, we may divide the parameter space (κ,ν) into two regions (dashed line in Fig. 5), in which $P(t)$ relaxes either as in the limit $\nu=0$ (under the line) or as in the $\nu=\infty$ one (over the line). In the latter case all four modes are important for the relaxation process, while in the former as ν increases two of them have less and less effect, since their damping rates increase with ν to infinity.

Knowing the eigenvalues of the problem one can write down the expression for $P(t)$. In the case of $b>0$ it is convenient to show it in compact form:

$$P(t)=\frac{1}{2}+\frac{1}{4\sqrt{b}}\exp(-(\kappa+\nu)t)[(\kappa\nu+\sqrt{b})C_1(t)$$
$$-(\kappa\nu-\sqrt{b})C_2(t)+(\nu(\kappa^2-\Delta_0^2+\sqrt{b})$$
$$+\kappa(\nu^2+\sqrt{b}))S_1(t)-(\nu(\kappa^2-\Delta_0^2-\sqrt{b})$$
$$+\kappa(\nu^2-\sqrt{b}))S_2(t)]. \tag{16}$$

If $0<a+2\sqrt{b}\equiv\Gamma_1^2$, then $C_1(t)=\cosh(\Gamma_1 t)$ and $S_1(t)=\sinh(\Gamma_1 t)/\Gamma_1$. If $0>a+2\sqrt{b}\equiv-\Omega_1^2$, then $C_1(t)=\cos(\Omega_1 t)$ and $S_1(t)=\sin(\Omega_1 t)/\Omega_1$. There are similar definitions for the quantity $a-2\sqrt{b}$ and the functions $C_2(t)$ and $S_2(t)$. When $b<0$, we have

$$P(t)=\frac{1}{2}+\frac{\Delta_0}{2\sqrt{-b}}\exp(-(\kappa+\nu)t)\sinh(\Gamma t)$$
$$\times\left[\frac{\Gamma\sqrt{-b}-(\nu^2-\Delta_1^2)\Omega}{\Gamma^2+\Omega^2}\cos(\Omega t)\right.$$
$$\left.+\left(\nu+\frac{(\nu^2-\Delta_1^2)\Gamma+\Omega\sqrt{-b}}{\Gamma^2+\Omega^2}\right)\sin(\Omega t)\right], \tag{17}$$

where we define $\pm\Gamma\pm i\Omega\equiv\pm\sqrt{a\pm 2i\sqrt{-b}}$.

The discussion about the phase diagram of possible types of eigenvalues suggests that, along the broken line in Fig. 5, one should expect the appearance of the resonancelike behavior which was found for $\kappa=0$. This is true for small κ, as shown in Fig. 6. Since an interaction with the surroundings damps the oscillations, for larger κ it starts to compete with the suppression of tunneling by barrier fluctuations. Consequently, for κ of the order of Δ_1, the resonance could no longer be seen.

However, for very large κ, in the region of localization caused by strong damping (Sec. III A), we can observe another effect of the enhancement of suppression of tunneling process (see Fig. 7), although it is rather tiny. As in the case of a static barrier, when κ is very large, two eigenvalues [Eq. (14)] are of the order of κ, while the other two become very small. Consequently $P(t)$ can be approximated as follows [24]:

$$P(t)\approx\frac{1}{2}+\frac{1}{2}[(1+\alpha_1-\alpha_2)\exp(-2(\kappa\alpha_1-\nu\alpha_3)t)$$
$$+\alpha_2\exp(-2(\nu+\kappa\alpha_1+\nu\alpha_3)t)], \tag{18}$$

where $\alpha_1=(\Delta_0^2+\Delta_1^2)/4\kappa^2$, $\alpha_2=4\Delta_0^2\Delta_1^2/4\kappa^2\nu^2$, and $\alpha_3=\alpha_2+\Delta_1^2/\kappa^2$. The reason for this effect is similar to that in the case of maximal suppression of tunneling for $\kappa\sim\Delta$ in a static barrier. For $\nu>1/\kappa$ only the first exponent in Eq. (18) contributes to the evolution of $P(t)$. Its decay rate approaches a maximum for $\nu=\Delta_0$, so in region of $\nu\sim\Delta_0$ we observe a maximal decay of $P(t)$. It follows from the form of α_3 that the effect could be seen for large values of Δ_1.

V. BIASED CASE

A. $\kappa=0$

The nonzero bias ($\epsilon\neq 0$) essentially complicates the problem, since all the components of $\vec{R}$ are dynamically coupled in Eq. (3a). However, for $\kappa=0$ one can reduce the problem of finding eigenvalues to the solution of a third-order polynomial. That is,

$$\lambda=-\nu\pm\sqrt{\nu^2+\mu}, \tag{19}$$

$$\mu^3+(\alpha_++\alpha_-)\mu^2+(\alpha_+\alpha_-+4\nu^2\alpha_0)\mu+4\nu^2\Delta_1\epsilon^2=0, \tag{20}$$

where $\alpha_\pm=(\Delta_0\pm\Delta_1)^2+\epsilon^2$ and $\alpha_0=(\Delta_0^2+\epsilon^2)$. Although Eq. (20) may in principle be solved analytically, nevertheless

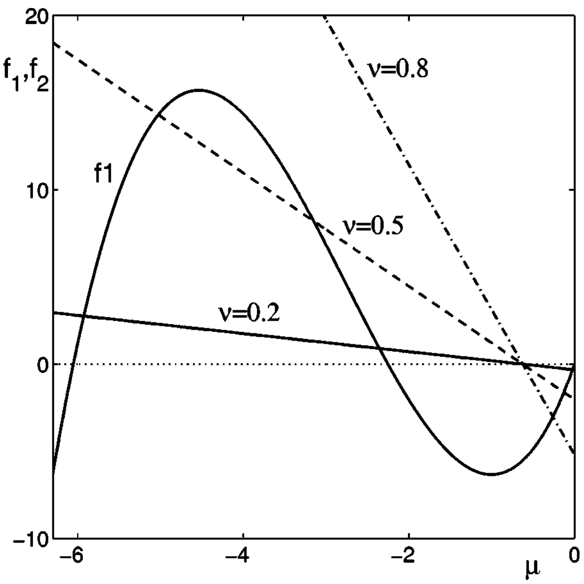

FIG. 8. Graphical solution of Eq. (20′) for a few values of ν: 0.2, 0.5, and 0.8. The other parameters are $\Delta_0=1.0$, $\Delta_1=0.95$, and $\epsilon=1.5$.

the result is too complicated for an exact analysis. Instead, from the form of Eq. (20) we may deduce some properties of its roots and then the way in which the eigenvalues depend on ν. To this end it is convenient to rewrite Eq. (20) by means of two auxiliary functions—a cubic function $f_1(\mu)=\mu(\mu+\alpha_+)(\mu+\alpha_-)$ and a linear one $f_2(\mu)=-4\nu^2(\alpha_0\mu+\Delta_1^2\epsilon^2)$:

$$f_1(\mu)-f_2(\mu)=0. \tag{20′}$$

Let us notice that ν interferes in Eq. (20′) through the slope of f_2, only. The zeros of both functions are unaffected by it. If $\nu=0$ then $f_2(\mu)$ vanishes, and zeros of $f_1(\mu)$ are the roots of Eq. (20). For a nonzero ν, real roots of Eq. (20) are given by intersection points of $f_1(\mu)$ and $f_2(\mu)$, as displayed in Fig. 8. We can see that as compared with the unbiased system a nonzero value of ϵ introduces quantitative changes only. As before, as ν increases the two, more negative, roots move closer to each other, eventually becoming a pair of complex roots. This corresponds to the case when ν becomes greater than Δ_1 in Sec. IV A. In addition, there is a third root of Eq. (20) which lies close to zero. It originates from the coupling with $\langle\sigma_x(t)\rangle$ and $\langle\sigma_x(t)\eta(t)\rangle$. For any ν it remains in the interval $(-\Delta_1^2\epsilon^2/\alpha_0,0)$. Thus for small Δ_1 we may estimate it perturbatively, leading to the following form of the smallest eigenvalue:

$$\lambda_1\approx-\frac{2\Delta_1^2\epsilon^2\nu}{\alpha_0(\alpha_0+4\nu^2)}. \tag{21}$$

In both limits of ν it vanishes but it reaches a minimum for $\nu=\sqrt{\alpha_0}/2$, with the value $\lambda_1=-\Delta_1^2\epsilon^2/(2\alpha_0^{3/2})$. From the other hand this value is the most negative for $\epsilon^2=2\Delta_0^2$.

The above characteristic features of eigenvalues are clearly reflected in the dynamics of the system (Fig. 9). For small ν, when we have two pairs of complex eigenvalues the situation is very similar to the unbiased case in Sec. IV A. The probability $P(t)$ oscillates with two frequencies close to $\alpha_\pm$, with a slightly damped amplitude. The only difference is that the levels detuning moves the center of oscillations toward greater probabilities. As ν reaches its bifurcation value where a pair of complex roots of Eq. (20) appears, we observe strong damping of $P(t)$. The mechanism is the same as discussed in Sec. IV A, although here the critical value of ν also depends on ϵ.

A further increase of ν leads to a new effect which is not present in the unbiased case. We observe strong damping for the value of ν for which a minimum of λ_1 appears. This means that, for this value of ν, the dynamics of $P(t)$ is affected very much by $\langle\sigma_x(t)\rangle$. Our estimation for small Δ_1 shows that, at this minimum, ν is of the order of the period of unperturbed oscillations, so it may be considered as a resonance between the characteristic time of static system and the rate of stochastic disturbance. Since λ_1 is a nonmonotonic function of ϵ, this resonance vanishes when the detuning of levels becomes too large. The first type of resonance also vanishes as ϵ increases, since it concerns the coherent part of $P(t)$ which decreases with detuning (Sec. III A). However, as can be seen in Fig. 10, after a suitably long time the second type of resonance causes a rapid decay of $P(t)$, while the first type does not.

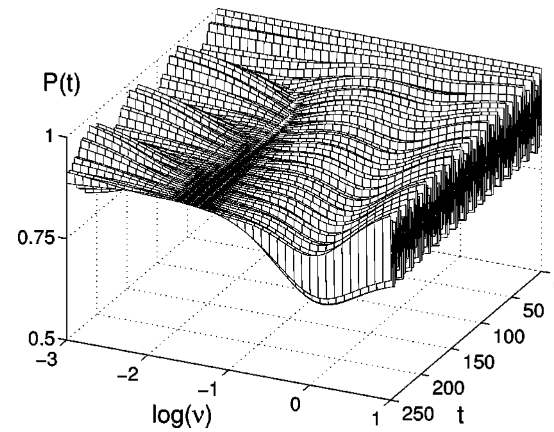

FIG. 9. Time evolution of $P(t)$ in dependence on the decimal logarithm of ν for the biased system ($\epsilon=2$) without an interaction with the environment ($\kappa=0$), for $\Delta_1=0.1$.

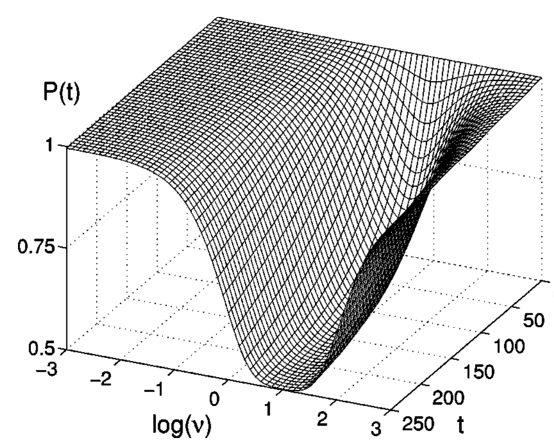

FIG. 10. The same as in Fig. 9, but for $\epsilon=20$ and $\Delta_1=0.95$.

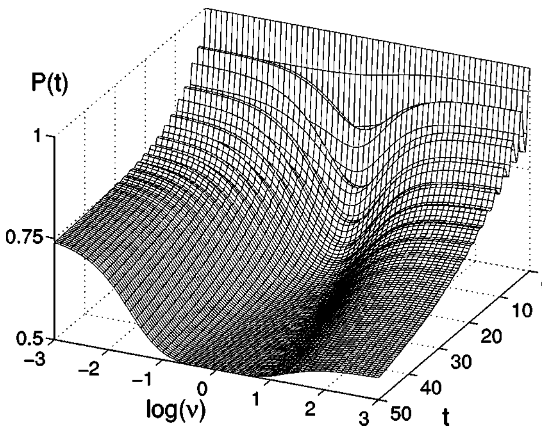

FIG. 11. Time evolution of $P(t)$ in the dependence on the decimal logarithm of ν for the biased case ($\epsilon=2$) with the interaction with the environment ($\kappa=0.1$), for $\Delta_1=0.95$.

B. $\kappa\neq 0$

The general case with both bias ($\epsilon\neq 0$) and interaction with the environment ($\kappa\neq 0$) cannot be solved analytically. However, it seems that the conclusions of the previous cases may be extrapolated to this case. As for the unbiased system, a nonzero κ introduces damping of the evolution, so the expected resonant effects become less clear (Fig. 11) or are even totally wiped out (Fig. 12).

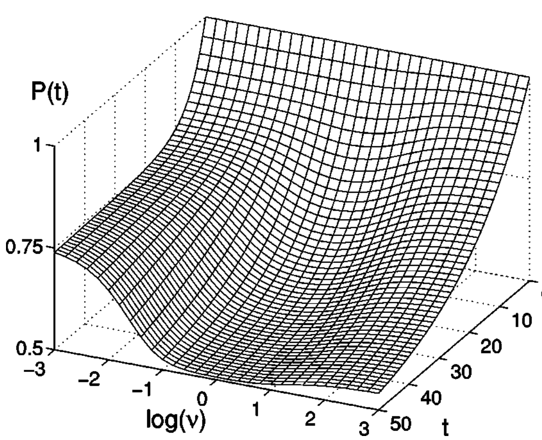

FIG. 12. The same as in Fig. 11, but for $\kappa=10$.

VI. CONCLUSIONS

In this paper we have investigated the evolution of a quantum particle placed in a double-well potential when the potential barrier is subjected to random fluctuations. The system has been approximated by a two-level model, while the modulation of the barrier has been given by a dichotomous noise $\eta(t)$. Additionally, the system has been allowed to interact with its environment. We have focused on the influence on the quantum tunneling process of the finite value of the jumping rate ν of $\eta(t)$. The most important finding of this research is the appearance of a resonant damping of the tunneling in both possible kinds of evolution: coherent and incoherent relaxation.

We have found two types of this resonance. The first one originates in the destructive interference of damped oscillations associated with the evolution in two possible configurations of the system. For ν of the order of the stochastic amplitude Δ_1 both these modes reach the same frequency, but with opposite phases that cause a very fast equilibration of the system. The most interesting fact is that the resonance refers solely to the parameters of the stochastic perturbation. Moreover, the characteristic time of this resonance may vary within a very large interval: from the mean value of the tunneling matrix element Δ_0 to the value of the damping constant κ resulting from the interaction with the surroundings. Hence one may exploit this mechanism to control the rate of suppression of tunneling phenomenon.

The second type of resonant damping occurs in a biased system, when σ_x is coupled with other dynamical variables. This interaction acts in a similar way to the thermal bath (see Sec. III A), and increases the rate of relaxation of the incoherent component of the evolution. The effect takes place for ν of the order of frequency of the coherent component (even when it is very small, cf. Fig. 9), so that the perturbation is in resonance with the system.

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[23] Similar expressions were obtained in Ref. [8], though without a systematic analysis of all possibilities.
[24] This approximation applies for ν much greater than $1/\kappa$ but smaller than κ.